WAMC12

Ryan Tang, Wesley Wu, and more

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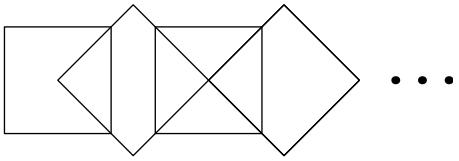
1 Problems

1. Ryan has a number. He multiplies it by 5, which increases the number by 1. He then multiplies it by 3. What is the number now?
(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{15}{4}$ (E) $\frac{25}{4}$

2. Planes are supposed to descend to the runway at 141 knots. One flight descends to the runway at 241 knots. One knot is equal to 1.15 miles per hour. How much faster than normal, in miles per hour, is that flight descending?
(A) 87 (B) 100 (C) 115 (D) 120 (E) 126

3. 2024 congruent squares with side length 2 are drawn in a repeating manner. Below are the leftmost four of the 2024 squares in the figure. Compute the area of the union of all the squares.



(A) 3036 (B) 6072 (C) 6073 (D) 8096 (E) 8097

4. If there exist positive integers $a, b \leq 5$ such that $6ab + 15a + 4b = 62$, find the value of $a + b$.
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

5. Wesley, Ryan, Alex, and Patrick are playing FTW for the first time in three months. In FTW, everyone must score a integer amount of points between 0 and 30, inclusive. Wesley scores 16 points in one game of FTW, while the second highest-scoring person has half that amount. How many scores in this game of FTW can there be? For example, one possible score is Wesley: 16, Ryan: 8, Alex: 5, Patrick: 0. Note that ties are allowed.
(A) 24 (B) 45 (C) 147 (D) 168 (E) 217

6. In a 4-digit number, the sum of the first two digits is 5, the sum of the middle two digits is 7, and the sum of the last two digits is 7. Find the sum of all possible values of this number.
(A) 10955 (B) 11190 (C) 14340 (D) 16260 (E) 16785

7. Suppose that X, O, K, S, R, and B are distinct digits. Suppose

$$\overline{XOOKS} + \overline{RBO} = 10434.$$

Compute $X + O + O + K + S + R + B + O$.
(A) 8 (B) 14 (C) 17 (D) 18 (E) 21

8. Positive integers a, b, c, d satisfy $a+b+c+d = 20$ and $a^3+b^3+c^3+d^3 = 530$. Find the value of $a^2 + b^2 + c^2 + d^2$.
(A) 100 (B) 102 (C) 104 (D) 275 (E) 400

9. In the expansion of $(2x^3 + x + 1) \cdot (30303x^6 + 4x^5 + x^4 + 3) \cdot (600x^5 + x^3 + 2x^2 + 4)$, what is the coefficient of the x^3 term?
(A) 6 (B) 11 (C) 18 (D) 27 (E) 33

10. A positive integer is a palindrome if it reads the same forwards and backwards. Call a positive integer an "emordnilap" if it reads the same when it is reflected about a horizontal line. For example, 18 is an emordnilap. How many five digit numbers are either palindromes or emordnilaps?
(A) 270 (B) 1620 (C) 1668 (D) 1860 (E) 2024

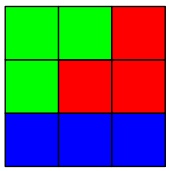
11. What is the probability that when rolling three 20−sided dice, the sum of the rolls has 10 divisors?
(A) $\frac{91}{8000}$ (B) $\frac{21}{1600}$ (D) $\frac{19}{800}$ (C) $\frac{91}{2000}$ (E) $\frac{21}{800}$

12. Find the sum of the digits of $116^2 - 85^2 + 12^2 + 9^2 + 4^2 + 3^2$.
(A) 7 (B) 10 (C) 15 (D) 16 (E) 19

13. Order these numbers from smallest to largest:
$A = (2024!)^{2024}$
$B = 2024^{2}!$
$C = 2024^{2024!}$
$D = (2024!)!$
(A) $ABCD$ (B) $BACD$ (C) $BDAC$ (D) $CDAB$ (E) $DCBA$

14. If the graph of all complex numbers z such that $k = \left(-\sqrt{2} + \sqrt{2}i\right)\left(\frac{z}{2} - \frac{1}{4}\right)$ has magnitude 1 and k is in the third quadrant forms an arc, what is the length of that arc?
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) π

15. How many ways are there to arrange the numbers 1 to 9 in this 3 by 3 grid so the numbers in the red squares sum to 7 and the numbers in the green squares sum to 22?



(A) 216 (B) 432 (C) 648 (D) 864 (E) 1296

16. Call a positive integer n *flippers* if n, when written in base 3, contains an equal amount of 0's, 1's, and 2's. How many *flippers* numbers are there from 1 to 3^9, inclusive?
(A) 1184 (B) 1440 (C) 1620 (D) 1776 (E) 1945

17. Let the infinite sum

$$\sum_{a=6}^{\infty}\left(\frac{a^{-1}}{a^2-3a+2}+\frac{a^{-1}}{a^2-1}+\frac{a^{-1}}{a^2+3a+2}\right)=k.$$

Then, k can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.
(A) 17 (B) 31 (C) 59 (D) 64 (E) 69

18. Bob and Bill go to a movie theater, independently. If Bob arrives there and Bill isn't there, he will wait for a maximum of 30 minutes for Bill to show up. If Bill goes to the movie theater, and he doesn't see Bob, Bill will wait for a maximum of 15 minutes for Bob to show up. If both of them are going to arrive at the movie theater between 12PM and 5PM, what is the probability that they will watch a movie together? (Note that if Bill shows up during Bob's waiting period, or vice versa, they will watch a movie together.)
(A) $\frac{23}{160}$ (B) $\frac{9}{50}$ (C) $\frac{1}{2}$ (D) $\frac{19}{36}$ (E) $\frac{17}{24}$

19. Ryan, Wesley, and Patrick are each uniformly randomly throwing one dart on a continuous number line from -10 to 10. Their score is defined as the distance from the number they hit to 3. What is the probability that the sum of their scores is less than 5?
(A) $\frac{4}{375}$ (B) $\frac{43}{3087}$ (C) $\frac{1}{48}$ (D) $\frac{11}{441}$ (E) $\frac{1}{24}$

20. Two circles with radius 19 and 7, respectively, are externally tangent to each other. A line ℓ passing through both circles intersects the larger circle at A and B and the smaller circle at C and D with B on AC and C on BD. If $AB=BC=CD$, compute the length of AD.
(A) 11 (B) 18 (C) 24 (D) 27 (E) 33

21. Compute $\left\lfloor \sum_{k=1}^{2024} \frac{1}{\sum_{j=1}^{k} \frac{10}{\sum_{x=1}^{j} x}} \right\rfloor$.
(A) 51 (B) 99 (C) 101 (D) 201 (E) 520

22. In isosceles trapezoid $ABCD$ with AB and CD parallel which has an incircle, the diagonal BD intersects the incircle of $ABCD$ at points P and Q, with P closer to D than Q is. Given that $DP = 3$ and $QB = 1$, the length of PQ can be expressed as $\sqrt{a + b\sqrt{c}} - d$, where a, b, c, and d are positive integers, and c does not divide any perfect squares. Compute $a + b + c + d$.
(A) 25 (B) 92 (C) 176 (D) 224 (E) 358

23. How many solutions in complex numbers are there to the system $\begin{cases} x^2y + y^2x = z \\ x^2z + z^2x = y \\ y^2z + z^2y = x? \end{cases}$
(A) 3 (B) 7 (C) 10 (D) 15 (E) 16

24. Triangle ABC with $AB < AC$ has circumcircle Γ. The altitude from A and the angle bisector of A intersect Γ at M and N, respectively. If $BM = MN = 4\sqrt{3} - 6$ and $\angle BMN = 150^\circ$, find $AB + AC$.
(A) 6 (B) $4\sqrt{3}$ (C) $6\sqrt{6}$ (D) 12 (E) $4\sqrt{3} + 6$

25. For how many ordered triples (a, b, c) of positive integers $1 \leq a, b, c \leq 30$ is $\frac{(a+b+c)^2}{a^2+b^2+c^2}$ noninteger?
(A) 26913 (B) 26925 (C) 26946 (D) 26970 (E) 27000